



DCU-003-0011005

Seat No. _____

B. Sc. (Sem. I) (CBCS) Examination

August - 2022

Mathematics : MATH – 01(A)

(Calculus) (Old Course)

Faculty Code : 003

Subject Code : 0011005

Time : $2\frac{1}{2}$ Hours]

[Total Marks : **70**

- Instructions :** (1) Attempt any five questions out of ten questions.
(2) Figures written on the right indicate marks of the question.

1 (a) Answer the following question briefly : **4**

- (1) Define : Bounded Set.
- (2) State Rolle's theorem.
- (3) Define strictly increasing function.
- (4) Write expansion of e^x in ascending power of x .

(b) Verify Rolle's theorem for the function **2**

$$f(x) = \tan x, \forall x \in \left[0, \frac{\pi}{4}\right].$$

(c) Expand $\sin x$ in ascending power of $\left(x - \frac{\pi}{2}\right)$. **3**

(d) State and prove Cauchy's Mean Value theorem. **5**

- 2** (a) Attempt the following questions briefly : **4**
- (1) Define : Bounded function.
 - (2) Set $A = \{\cos x/x \in R\}$ then $LUB(A) = \underline{\hspace{2cm}}$ and $GLB A = \underline{\hspace{2cm}}$.
 - (3) In alternative form of Cauchy's mean value theorem in which interval θ lies?
 - (4) State the Maclaurin's series expansion of $\sin x$.
- (b) Show that the function $2 - 3x + 6x^2 - 4x^3$ is strictly **2**
decreasing in every interval.
- (c) For $f(x) = x^x, g(x) = x \log x$ where $a < c < b$ show that **3**
 $b^b - a^a = c^c [b \log b - a \log a]$.
- (d) Find coefficient of x^4 in the expansion of $\log \cos x$ **5**
where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.
- 3** (a) Answer the following questions briefly : **4**
- (1) Evaluate : $\lim_{x \rightarrow 0} \frac{3x}{\sin 2x}$.
 - (2) Define order of differential equation.
 - (3) Solve the differential equation $xdy + ydx = 0$.
 - (4) State fundamental indeterminate forms.
- (b) Solve : $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$. **2**
- (c) If $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3} = b$ then find value of a and b . **3**
- (d) Find the solution of linear differential equation **5**
 $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x .

4 (a) Answer the following questions briefly : 4

(1) Evaluate : $\lim_{x \rightarrow 0} \frac{3^x - 4^x}{x}$.

(2) State any five indeterminate forms.

(3) State the degree of differential equation

$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}.$$

(4) Find the general solution of differential equation

$$\sqrt{1-x^2} dy + \sqrt{1-y^2} dx = 0.$$

(b) Evaluate : $\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1 + \log x}$. 2

(c) Solve : $\cos^2 x \frac{dy}{dx} + y = \tan x$. 3

(d) Evaluate : $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x}$. 5

5 (a) Answer the following questions briefly : 4

(1) State the general form of Bernoulli's differential equation.

(2) State the necessary and sufficient condition for the differential equation $M(x, y) dx + N(x, y) dy = 0$ to be exact.

(3) Solve : $y - px = p^2$.

(4) Solve : $yp - x = 0$.

(b) Solve : $y = xp - p^2 + \log p$. **2**

(c) Solve : $(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)dy = 0$. **3**

(d) Solve : $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ **5**

6 (a) Answer the following question briefly : **4**

(1) State the general form of Lagrange's differential equation.

(2) Find the integrating factor of differential equation

$$\frac{dy}{dx} = x + y.$$

(3) Solve : $py = p^2x + m$.

(4) Solve : $p^2 - p - 6 = 0$.

(b) Find the solution of the equation **2**

$$(x^2 - ay)dx + (y^2 - ax)dy = 0.$$

(c) Solve : $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. **3**

(d) Solve : $p^2 - 6px + 3y = 0$ where, $p = \frac{dy}{dx}$. **5**

7 (a) Answer the following questions briefly : **4**

(1) State the general form of Linear Differential equation of n^{th} order.

(2) Write the auxiliary equation of $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$.

(3) Evaluate : $\frac{1}{D-3}e^{5x}$.

(4) Evaluate : $\frac{1}{D}x^2$.

(b) Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$. **2**

(c) Find $\frac{1}{D-3}\sin 2x$. **3**

(d) Solve : $\frac{d^2y}{dx^2} + 4y = \sin 2x$. **5**

8 (a) Answer the following questions briefly : **4**

(1) Find the C.F. (complimentary function) of

$$(D^2 + 9)y = \sin x.$$

(2) Evaluate : $\frac{1}{D^3}3x^3$.

(3) Evaluate : $\frac{1}{D^2 + 7}\sin 2x$.

(4) Evaluate : $\frac{1}{(D-2)^3}e^{2x}$.

(b) Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$. **2**

(c) Solve : $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 2y = e^{5x}$. **3**

(d) Solve : $(D^2 - 2D + 1)y = x^2e^{3x}$. **5**

9 (a) Answer the following questions : **4**

(1) Find C.F. of $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$.

(2) Find C.F. of $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$.

(3) Find P.I. (particular integral) of

$$(x^2D^2 + 5xD + 4)y = x^4.$$

(4) Evaluate : $\frac{1}{(D-1)^2}e^x$.

(b) Solve : $(x^2D^2 + xD - 9)y = 0$. **2**

(c) Solve : $(x^3D^2 + x^2D)y = x^6 + x^4 + 2x$ where $D = \frac{d}{dx}$. **3**

(d) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2\log x$. **5**

10 (a) Answer the following questions briefly : **4**

(1) Evaluate : $\frac{1}{D+1}2x$.

(2) Solve : $(4x^2D^2 + 1)y = 0$ where $D = \frac{d}{dx}$.

(3) Find P.I. of $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = \frac{12 \log x}{x^2}$.

(4) Find P.I. of $x^2 \frac{d^2y}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$.

(b) Solve : $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$. **2**

(c) Solve : $\frac{d^2y}{dx^2} = \frac{6y}{x^2}$. **3**

(d) Solve : $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$. **5**
